12.1 Explore Solids

Goal • Identify solids.

Your Notes

VOCABULARY

Polyhedron A polyhedron is a solid that is bounded by polygons that enclose a single region of space.

Face The faces of a polyhedron are polygons.

Edge An edge of a polyhedron is a line segment formed by the intersection of two faces.

Vertex A vertex of a polyhedron is a point where three or more edges meet.

Base A base is a polygon that is used to name the polyhedron.

Regular polyhedron A regular polyhedron is a polyhedron whose faces are all congruent regular polygons.

Convex polyhedron A convex polyhedron is a polyhedron such that any two points on its surface can be connected by a line segment that lies entirely inside or on the polyhedron.

Platonic solids A Platonic solid is one of five regular polyhedra: a regular tetrahedron, a cube, a regular octahedron, a regular dodecahedron, and a regular icosahedron.

Notice that the names of four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." Sometimes a cube is called a regular *hexahedron*.

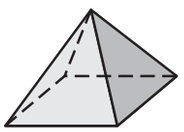
Cross section A cross section is the intersection of a plane and a solid.

Your Notes

TYPES OF SOLIDS

Polyhedra

Prism



Pyramid

Not Polyhedra

Cylinder

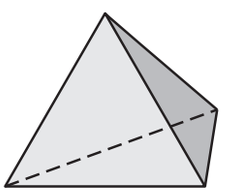
Cone

Sphere

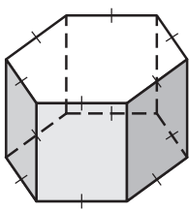
Example 1 ***Identify and name polyhedra***

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

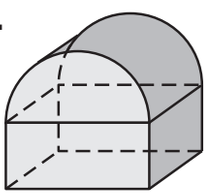
a.



b.



c.



Solution

a. This is a polyhedron. It has 4 faces so it is a tetrahedron. It has 4 vertices and 6 edges.

b. This is a polyhedron. The two bases are congruent hexagons, so it is a hexagonal prism. It has 8 faces, 12 vertices, and 18 edges.

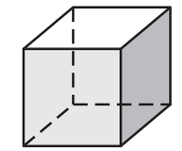
c. This is not a polyhedron. The solid has a curved surface.

Your Notes

THEOREM 12.1: EULER'S THEOREM

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula

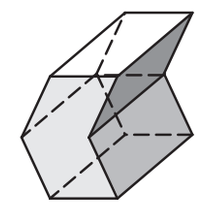
$F + V = E +$ 2 .



$F = 6$, $V = 8$, $E = 12$
$6 + 8 = 12 + 2$

Example 2 *Use Euler's Theorem*

Find the number of faces, vertices, and edges of the polyhedron shown. Check your answers using Euler's Theorem.



The polyhedron has 8 faces, 12 vertices, and 18 edges.

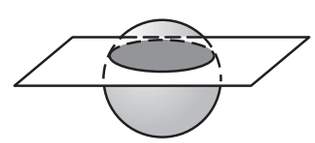
Use Euler's Theorem to check.

$F + V = E + 2$	**Euler's Theorem**
8 + 12 = 18 + 2	**Substitute.**
20 = 20	**Check.**

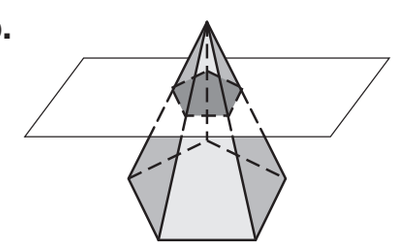
Example 3 *Describe cross sections*

Describe the shape formed by the intersection of the plane and the solid.

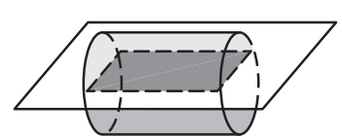
a.



b.



c.



Solution

a. The cross section is a circle .

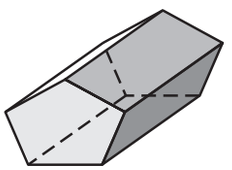
b. The cross section is a pentagon .

c. The cross section is a rectangle .

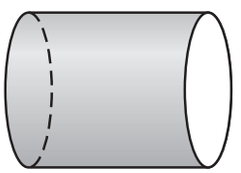
Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

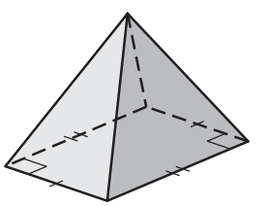
In Exercises 1–3, tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

1. 

This is a polyhedron. Its bases are congruent pentagons, so it is a pentagonal prism. It has 7 faces, 10 vertices, and 15 edges.

2. 

This is not a polyhedron. The solid has a curved surface.

3. 

This is a polyhedron. Its base is a rectangle, so it is a rectangular pyramid. It has 5 faces, 5 vertices, and 8 edges.

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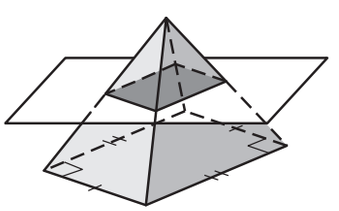
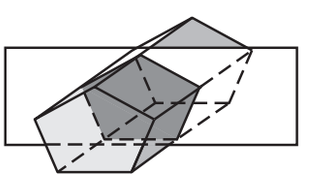
4. Is it possible for a polyhedron to have 16 faces, 34 vertices, and 50 edges? *Explain*.

No; Using Euler's Theorem $F + V = E + 2$, $16 + 34 \neq 50 + 2$.

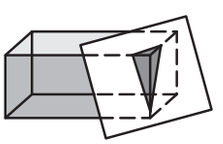
In Exercises 5–7, *describe* the shape formed by the intersection of the plane and the solid.

5. 

rectangle

6. 

pentagon

7. 

triangle

Homework