

12.3

Surface Area of Pyramids and Cones

Goal • Find the surface areas of pyramids and cones.

Your Notes

Pyramids are classified by the shapes of their bases.

VOCABULARY

Pyramid A pyramid is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

Vertex of a pyramid The vertex of a pyramid is the common vertex of the lateral faces of a pyramid.

Regular pyramid A regular pyramid has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles.

Slant height The slant height of a regular pyramid is the height of a lateral face of the regular pyramid.

Cone A cone has a circular base and a vertex that is not in the same plane as the base.

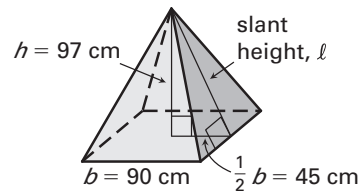
Vertex of a cone The vertex of a cone is the point on the cone that is located at a perpendicular distance from the base, called the height of the cone.

Right cone In a right cone, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

Lateral surface The lateral surface of a cone consists of all segments that connect the vertex with points on the base edge.

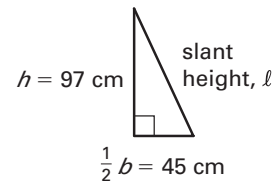
Example 1 Find the area of a lateral face of a pyramid

Find the area of each lateral face of the regular square pyramid.



Solution

Use the Pythagorean Theorem to find the slant height l .



$$l^2 = h^2 + \left(\frac{1}{2}b\right)^2$$

Write formula.

$$l^2 = 97^2 + 45^2$$

Substitute for h and $\frac{1}{2}b$.

$$l^2 = 11,434$$

Simplify.

$$l = \pm\sqrt{11,434}$$

Take the square root of each side.

$$l \approx 106.93$$

Find the positive square root.

Find the area of each triangular face.

$$A = \frac{1}{2}bl$$

Write formula.

$$\approx \frac{1}{2}(90)(106.93)$$

Substitute for b and l .

$$\approx 4811.85$$

Simplify.

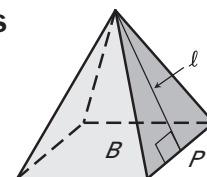
The area of each lateral face is about 4811.85 square centimeters.

THEOREM 12.4: SURFACE AREA OF A REGULAR PYRAMID

The surface area S of a regular pyramid is

$$S = B + \frac{1}{2}Pl,$$

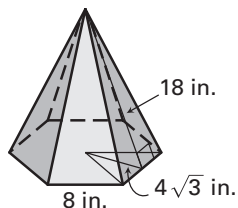
where B is the area of the base, P is the perimeter of the base, and l is the slant height.



Your Notes

Example 2 Find the surface area of a pyramid

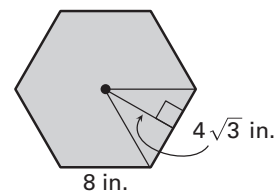
Find the surface area of the regular hexagonal pyramid.



Remember, the *apothem* of a polygon is the distance from the center to any side of the polygon.

Solution

First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $4\sqrt{3}$ inches and the perimeter P is $6 \cdot 8 = 48$ inches.



So, the area of the base B is $\frac{1}{2}(4\sqrt{3})(48) = 96\sqrt{3}$ square inches. Then, find the surface area.

$$\begin{aligned}
 S &= B + \frac{1}{2}Pl && \text{Surface area of regular pyramid} \\
 &= 96\sqrt{3} + \frac{1}{2}(48)(18) && \text{Substitute.} \\
 &= 96\sqrt{3} + 432 && \text{Simplify.} \\
 &\approx 598.3 && \text{Use a calculator.}
 \end{aligned}$$

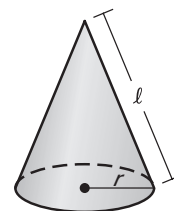
The surface area of the regular hexagonal pyramid is about 598.3 square inches.

THEOREM 12.5: SURFACE AREA OF A RIGHT CONE

The surface area S of a right cone is

$$S = B + \frac{1}{2}Cl = \pi r^2 + \pi r\ell,$$

where B is area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.



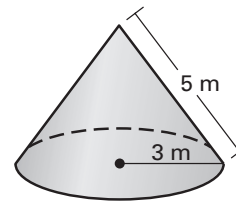
Your Notes

Example 3 Find the surface area of a right cone

Find the surface area of the right cone.

Solution

The base has a radius of 3 meters, and the cone has a slant height of 5 meters.



$$S = \pi r^2 + \pi r l$$

$$= \pi(\underline{3})^2 + \pi(\underline{3})(\underline{5})$$

$$= \underline{9} \pi + \underline{15} \pi$$

$$= \underline{24} \pi$$

$$\approx \underline{75.4}$$

Surface area of right cone

Substitute.

Simplify.

Add.

Use a calculator.

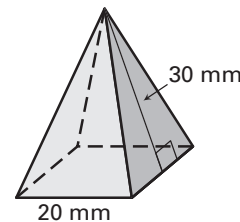
The surface area of the right cone is about 75.4 square meters.

Checkpoint Complete the following exercises.

1. Find (a) the area of each lateral face and (b) the surface area of the regular square pyramid.

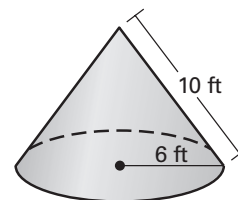
a. 300 mm^2

b. 1600 mm^2



2. Find the surface area of the right cone. Round your answer to two decimal places.

301.59 ft^2



Homework