12.4 Volume of Prisms and Cylinders

Goal • Find volumes of prisms and cylinders.

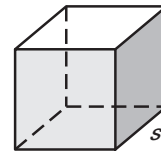*Your Notes*

VOCABULARY

Volume The volume of a solid is the number of cubic units contained in its interior.

POSTULATE 27: VOLUME OF A CUBE POSTULATE

The volume of a cube is the cube of the length of its side.

$$V = s^3$$

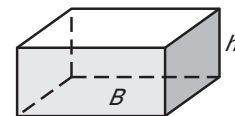---

POSTULATE 28: VOLUME CONGRUENCE POSTULATE

If two polyhedra are congruent, then they have the same volume .

POSTULATE 29: VOLUME ADDITION POSTULATE

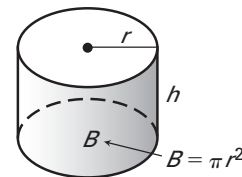The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

THEOREM 12.6: VOLUME OF A PRISM

The volume V of a prism is $V = Bh$, where B is the area of the base and h is the height.
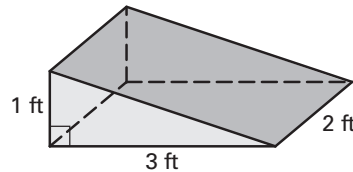

THEOREM 12.7: VOLUME OF A CYLINDER

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.
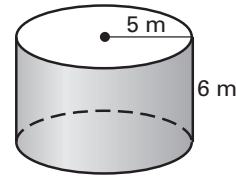
Example 1 *Find volumes of prisms and cylinders*

Find the volume of the solid.

a. Right triangular prism

b. Right cylinder

Solution

a. The area of the base is $\frac{1}{2}(\underline{\ 1\ })(\underline{\ 3\ }) = \underline{\frac{3}{2}}$ ft^2 and $h = 2$ ft.
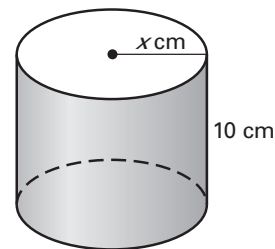
$V = Bh = \underline{\frac{3}{2}} \cdot \underline{\ 2\ } = \underline{\ 3\ }$ ft^3

b. The area of the base is $\pi \cdot \underline{\ 5\ }^2$, or $\underline{\ 25\ }\ \pi$ m^2. Use $h = 6$ to find the volume.

$V = Bh = \underline{\ 25\ }\ \pi(\underline{\ 6\ }) = \underline{\ 150\ }\ \pi \approx \underline{\ 471.24\ }$ m^3

Example 2 *Use volume of a right cylinder*

The volume of the right cylinder shown is **1253 cubic centimeters.** Find the value of *x*.

Solution

The area of the base is πx^2 square meters.

$V = Bh$ **Formula for volume of a cylinder**

$\underline{\ 1253\ } = \pi x^2(\underline{\ 10\ })$ **Substitute.**

$\underline{\ 1253\ } = \underline{\ 10\ }\ \pi x^2$ **Rewrite.**

$\dfrac{\boxed{1253}}{\boxed{10}\ \pi} = x^2$ **Divide each side by $\underline{\ 10\ }\ \pi$.**

$\underline{\ 39.88\ } \approx x^2$ **Simplify.**

$\underline{\ 6.32\ } \approx x$ **Find the positive square root.**

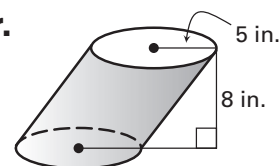The radius of the cylinder is about $\underline{\ 6.32\ }$ centimeters.

THEOREM 12.8: CAVALIERI'S PRINCIPLE

If two solids have the same height and the same cross-sectional area at every level, then they have the same __volume__.

Example 3   *Find the volume of an oblique cylinder*

Find the volume of the oblique cylinder.

5 in.

8 in.

Solution

> Cavalieri's Principle tells you that the volume formulas work for oblique prisms and cylinders.

Cavalieri's Principle allows you to use Theorem 12.7 to find the volume of the oblique cylinder.

$V = \pi r^2 h$ **Formula for volume of a cylinder**

$= \pi(\underline{\ 5\ }^2)(\underline{\ 8\ })$ **Substitute.**

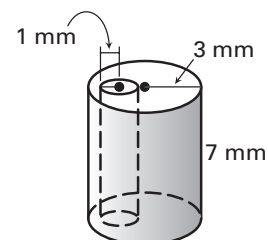$= \underline{\ 200\ }\ \pi$ **Simplify.**

$\approx \underline{\ 628.32\ }$ **Use a calculator.**

The volume of the oblique cylinder is about __628.32__ in.^{3}

Example 4   *Find the volume of a composite solid*

Find the volume of the solid.

1 mm

3 mm

7 mm

Solution

The area of the base *B* can be found by subtracting the area of the small circle from the area of the large circle.

B = Area of large circle − Area of small circle

$= \pi(\underline{\ 3\ }^2) - \pi(\underline{\ 1\ }^2) = \underline{\ 8\ }\ \pi \approx \underline{\ 25.13\ }$ mm^2

Use the formula for volume of a cylinder.

$V = Bh$ **Formula for volume of a cylinder**

$= (\underline{\ 25.13\ })(\underline{\ 7\ })$ **Substitute.**

$= \underline{\ 175.91\ }$ **Use a calculator.**
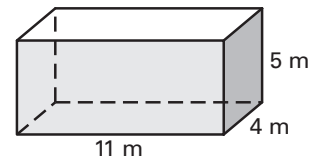
The volume of the solid is about __175.91__ cubic millimeters.

✔ *Checkpoint* **Complete the following exercises.**

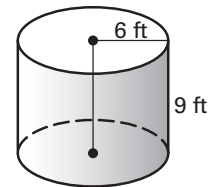**In Exercises 1 and 2, find the volume of the solid. Round your answer to two decimal places, if necessary.**
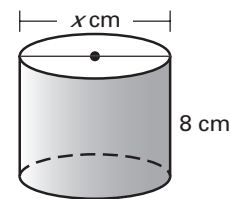
1. Right rectangular prism

220 m³

5 m

4 m

11 m

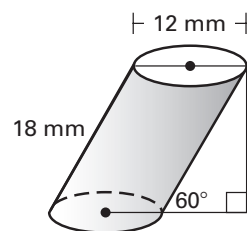**2.** Right cylinder

1017.88 ft³

6 ft

9 ft

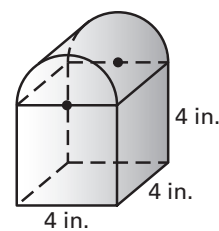**3.** The volume of the right cylinder is 200π cubic centimeters. Find the value of x.

10 cm

x cm

8 cm

In Exercises 4 and 5, find the volume of the solid. Round your answer to two decimal places.

4. Oblique cylinder

12 mm

18 mm

60°

1763.01 mm³

5. Composite solid

4 in.

4 in.

4 in.

89.13 in.³

Homework