

12.5

Volume of Pyramids and Cones

Goal • Find volumes of pyramids and cones.

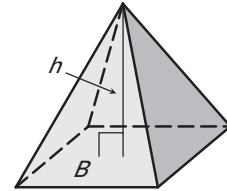
Your Notes

THEOREM 12.9: VOLUME OF A PYRAMID

The volume V of a pyramid is

$$V = \frac{1}{3} Bh,$$

where B is the area of the base and h is the height.



Example 1 Find the volume of a pyramid

Find the volume of the pyramid with the regular base.

Solution

First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $2\sqrt{3}$ inches and the perimeter P is $6 \cdot 4 = 24$ inches.

So, the area of the base B is $\frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$ in.².

Then, find the volume.

$$V = \frac{1}{3} Bh$$

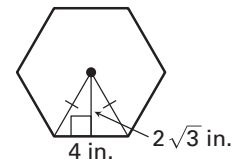
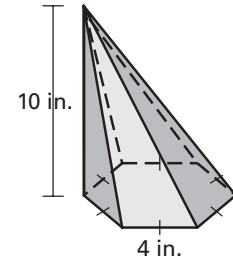
Formula for volume of a pyramid

$$= \frac{1}{3} 24\sqrt{3} (10) \quad \text{Substitute.}$$

$$= 80\sqrt{3} \quad \text{Simplify.}$$

$$\approx 138.56 \quad \text{Use a calculator.}$$

The volume of the pyramid is about 138.56 cubic inches.



The formula given in Theorem 12.9 applies to both right and oblique pyramids. This follows from Cavalieri's Principle.

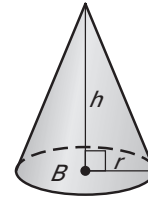
Your Notes

THEOREM 12.10: VOLUME OF A CONE

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2h,$$

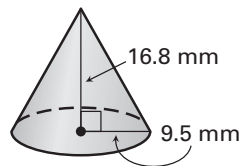
where B is area of the base, h is the height, and r is the radius of the base.



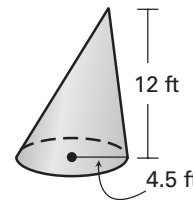
Example 2 Find volumes of cones

Find the volume of the cone.

a. Right cone



b. Oblique cone



Solution

a. Use the formula for volume of a cone.

$$V = \frac{1}{3}\pi r^2h$$

Formula for volume of a cone

$$= \frac{1}{3}\pi(9.5^2)(16.8)$$

Substitute.

$$= 505.4\pi$$

Simplify.

$$\approx 1587.8$$

Use a calculator.

The volume of the right cone is about 1587.8 mm^3 .

b. Use the formula for volume of a cone.

$$V = \frac{1}{3}\pi r^2h$$

Formula for volume of a cone

$$= \frac{1}{3}\pi(4.5^2)(12)$$

Substitute.

$$= 81\pi$$

Simplify.

$$\approx 254.5$$

Use a calculator.

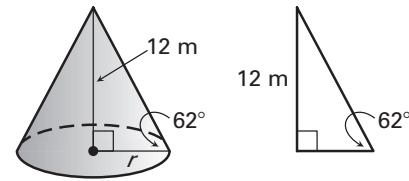
The volume of the oblique cone is about 254.5 ft^3 .

The formula given in Theorem 12.10 applies to both right and oblique cones. This follows from Cavalieri's Principle.

Your Notes

Example 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.



Solution

To find the radius r of the base, use trigonometry.

$$\tan 62^\circ = \frac{\text{opp.}}{\text{adj.}} \quad \text{Write ratio.}$$

$$\tan 62^\circ = \frac{12}{r} \quad \text{Substitute.}$$

$$r = \frac{12}{\tan 62^\circ} \approx 6.38 \quad \text{Solve for } r.$$

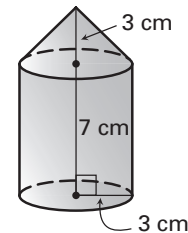
Use the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(6.38^2)(12) \approx 511.5$$

The volume of the cone is about 511.5 cubic meters.

Example 4 Find volume of a composite solid

Find the volume of the solid shown. The cone and the cylinder are right.



Solution

Volume of solid = Volume of cylinder + Volume of cone

$$= \pi r^2 h + \frac{1}{3} \pi r^2 h$$

$$= \pi(3^2)(7) + \frac{1}{3}\pi(3^2)(3) \quad \text{Substitute.}$$

$$= 63\pi + 9\pi \quad \text{Simplify.}$$

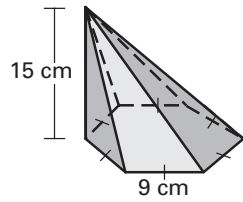
$$= 226.19 \quad \text{Use a calculator.}$$

The volume of the solid is about 226.19 cubic centimeters.

Your Notes

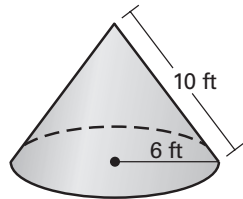
✓ **Checkpoint** Find the volume of the solid. Round your answer to two decimal places.

1. Pyramid with regular base



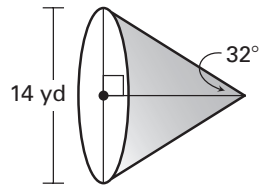
1052.22 cm³

2. Right cone



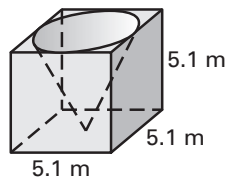
301.59 ft³

3. Right cone



574.70 yd³

4. Composite solid



97.92 m³

Homework