Slope of a Line

Slope = m

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Finding the slope from a graph.

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$\frac{\text{up/down}}{\text{left/right}}$

$\frac{+\text{ up}}{+\text{ right}}$ = positive slope

$\frac{-\text{ down}}{-\text{ left}}$ = positive slope

$\frac{+\text{ up}}{-\text{ left}}$ = negative slope

$\frac{-\text{ down}}{+\text{ right}}$ = negative slope

$\frac{0 \text{ No Rise}}{\text{left/right}}$ = 0 slope

$\frac{+/- \text{ up/down}}{\text{No RUN } 0}$ = undefined

$\longleftrightarrow$ $m = 0$, $y = \#$

horizontal line

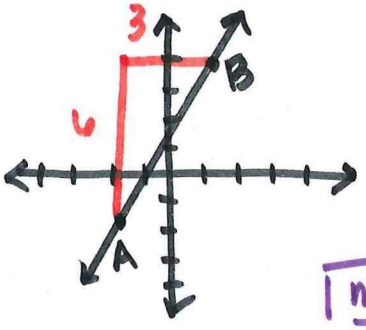
$m = \frac{0}{\#}$

m = undefined

vertical line

$m = \frac{\#}{0}$ $x = \#$

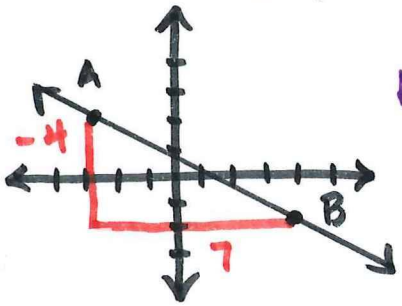
Start at the left most point on a graph.



Rise = 6
Run = 3

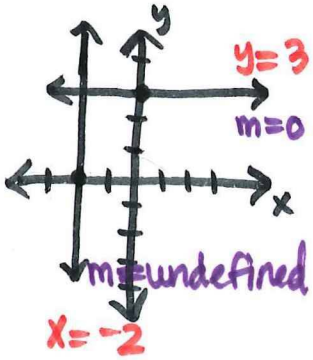
$m = \frac{6}{3}$

$m = 2$ or $\frac{2}{1}$



Rise = -4
Run = 7

$m = \frac{-4}{7}$



Finding slope from Two Points

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x, y)$$

① Label your points.

$(5, -3)$, $(-8, -12)$ — $x_1\ y_1$, $x_2\ y_2$

② Substitute into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-12 + +3}{-8 + -5} = \frac{-9}{-13}$$

③ Simplify (if possible).

$$\boxed{m = \frac{9}{13}}$$

Examples.

$(21, 10)$, $(8, 15)$ — $x_1\ y_1$, $x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{15 - 10}{8 - 21} = \frac{5}{-13}$$

$$\boxed{m = \frac{-5}{13}}$$

$(5, -1)$, $(12, -1)$ — $x_1\ y_1$, $x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1 + +1}{12 - 5} = \frac{0}{7}$$

$\boxed{m = 0}$ $y = -1$

$(8, -4)$, $(8, 0)$ — $x_1\ y_1$, $x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 + +4}{8 - 8} = \frac{4}{0}$$

$\boxed{m = \text{undefined}}$ $x = 8$

Finding the slope from an Equation

m = slope

Slope-Intercept form: $y = mx + b$

m: slope

b: y-intercept, crosses y-axis, $(0, b)$

* The slope is the coefficient of x: number in front of x
DO NOT INCLUDE x in your answer

example: $y = 3x - 8$ $\qquad$ $y = -\frac{1}{2}x + 12$

what is the slope of the line? $\qquad$ $m = 3$ $\qquad$ $m = -\frac{1}{2}$

Standard Form: $Ax + By = C$

Solve for y: $Ax + By = C$

$-Ax \qquad -Ax$

$$By = -Ax + C$$

$$\frac{By}{B} = \frac{-Ax + C}{B}$$

$$\boxed{y = \frac{-A}{B}x + \frac{C}{B}}$$

$$m = \frac{-A}{B} \qquad b = \frac{C}{B}$$

examples:

$$2x - 8y = 4$$
$$-2x \qquad -2x$$

$$\frac{-8y}{-8} = \frac{-2x + 4}{-8}$$

$$y = \frac{-2}{-8}x + \frac{4}{-8}$$

$$y = \frac{1}{4}x + \frac{-1}{2}$$

$$m = \frac{1}{4} \quad b = \frac{-1}{2}$$

Graphing Lines

Slope-intercept form: $y = mx + b$

- m = slope, $m = \frac{\text{rise}}{\text{run}}$
- b = y-intercept, crosses y-axis $(0, b)$
  * starting point

step 1 Plot your y-intercept $(0, b)$

step 2 Plot additional points using the slope $(\frac{\text{rise}}{\text{run}})$ from your y-intercept $(0, b)$

Standard form: $Ax + By = C$

option 1

Re-write the standard form into slope-intercept form and then follow steps 1 and 2 from above.

example:

$2x - 3y = 15$

$-2x \qquad -2x$

$-3y = -2x + 15$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{15}{-3}$

$y = \frac{2}{3}x - 5$

$(0, 5)$

$m = 2/3$

option 2

Find/solve for the x and y-intercepts from the standard form. Then plot $(x, 0)$ and $(0, y)$ so that you can connect the 2 points and find the slope from there.

x-intercept: plug in 0 for y and solve for x. $(x, 0)$

y-intercept: plug in 0 for x and solve for y. $(0, y)$

$2x - 3y = 15$

x-intercept $(x, 0)$

$2x - 3(0) = 15$

$2x = 15$

$x = 7.5$

$(7.5, 0)$

crosses the x-axis

y-intercept $(0, y)$

$2(0) - 3y = 15$

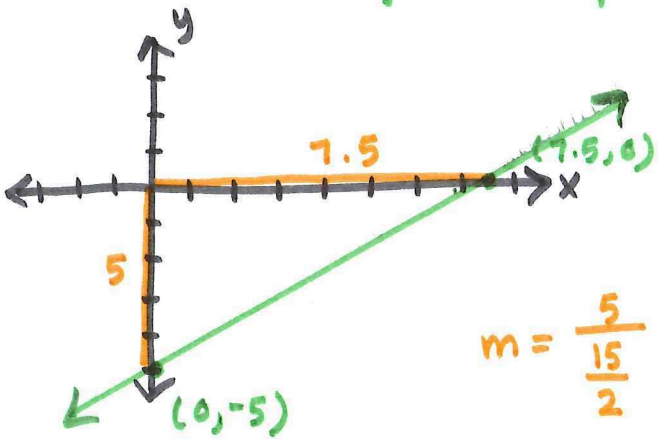
$-3y = 15$

$y = -5$

$(0, -5)$

crosses the y-axis

① Plot the x and y-intercepts on a graph



② connect the 2 points

③ Find the slope by using $\frac{\text{rise}}{\text{run}}$

$m = \frac{5}{\frac{15}{2}}$

$5 \div \frac{15}{2}$

$5 \times \frac{2}{15} = \frac{10}{15}$

$= \frac{2}{3}$

$\boxed{m = \frac{2}{3}}$

④ write the equation in slope-intercept form.

$m = \frac{2}{3}$ $b = -5$

$y = mx + b$

$\boxed{y = \frac{2}{3}x - 5}$

Writing Linear Equations

slope-intercept form: $y = mx + b$

standard form: $Ax + By = C$

point-slope form: $y - y_1 = m(x - x_1)$ point: (x_1, y_1), slope: m

write the slope-intercept form of the equation.

1. $-5x + 2y = 22$ standard

$+5x \quad +5x$

$\frac{2y}{2} = \frac{5x + 22}{2}$

$y = \frac{5}{2}x + 11$

2. through: (1, 8) (point), slope = 4

option 1 use $y = mx + b$

(1, 8) $m = 4$ solve for b

$8 = 4(1) + b$

$8 = 4 + b$

$-4 \quad -4$

$b = 4$

$y = 4x + 4$

option 2 use $y - y_1 = m(x - x_1)$

(1, 8) $m = 4$

$y - 8 = 4(x - 1)$

$y - 8 = 4x - 4$

$+8 \quad +8$

$y = 4x + 4$

3. through: (4, 2) (point), parallel to $y = \frac{-3}{4}x - 5$ (same slope)

$m = \frac{-3}{4}$

$y - 2 = \frac{-3}{4}(x - 4)$

$\frac{-3}{4}\left(\frac{-4}{1}\right) = \frac{12}{4}$

$y - 2 = \frac{-3}{4}x + 3$

$+2 \quad +2$

$y = \frac{-3}{4}x + 5$

4. through: (2, 4) (point), perpendicular to $y = \frac{-1}{5}x + 7$ (find the opposite reciprical)

$m = 5$

$y - 4 = 5(x - 2)$

$y - 4 = 5x - 10$

$+4 \quad +4$

$y = 5x - 6$