

Ratios, Proportions and Geometric Means

Vocabulary	Definition	Example
RATIO of A to B	If a and b are two numbers or quantities and $b \neq 0$, then the ratio of a to b is a/b . The ratio of a to b can also be written as $a : b$.	$\frac{a}{b}$ $a : b$ $a \text{ to } b$ $\frac{3}{5}$
PROPORTIONS	An equation that states that two ratios are equal is called a proportion.	$\frac{3}{5} = \frac{6}{10}$
MEANS and EXTREMES	The numbers b and c are the means of the proportion. The numbers a and d are the extremes of the proportion.	$\begin{array}{ccccc} \text{extreme} & \rightarrow & \frac{a}{b} & = & \frac{c}{d} & \leftarrow & \text{mean} \\ & & \text{mean} & & \text{extreme} & & \end{array}$
PROPERTIES OF PROPORTIONS		
CROSS PRODUCTS PROPERTY	In a proportion, the product of the extremes equals the product of the means.	<p>if $\frac{a}{b} = \frac{c}{d}$ then $a(d) = b(c)$</p> <p>if $\frac{3}{5} = \frac{6}{10}$ then $3(10) = 5(6)$ $30 = 30$.</p> <div style="text-align: right; color: green;"> $\frac{2}{7} = \frac{x}{14}$ $\xrightarrow{\times 2}$ $\frac{2}{7} = \frac{x}{14}$ $\xrightarrow{\times 7}$ $2 = \frac{x}{2}$ $7x = 28$ $\underline{7x = 28}$ $x = 4$ </div>

<p>RECIPROCAL PROPERTY</p>	<p>It two ratios are equal, then their reciprocals are also equal.</p> $\frac{1}{2} \quad \frac{2}{1} \quad 3 \quad \frac{1}{3}$	<p>If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$.</p> <p>If $\frac{3}{5} = \frac{6}{10}$ then $\frac{5}{3} = \frac{10}{6}$.</p>																
<p>INTERCHANGE the MEANS</p>	<p>If you interchange the means of a proportion, then you form another true proportion.</p>	<p>If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$.</p> <p>If $\frac{3}{5} = \frac{6}{10}$ then $\frac{3}{6} = \frac{5}{10} \times 2$ $\frac{1}{2} = \frac{1}{2}$</p>																
<p>DENOMINATOR ADDITION</p>	<p>In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion.</p>	<p>If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$.</p> <p>If $\frac{3}{5} = \frac{6}{10}$ then $\frac{3+5}{5} = \frac{6+10}{10}$ $\frac{8}{5} = \frac{16}{10}$</p>																
<p>GEOMETRIC MEAN</p>	<p>The geometric mean of two positive numbers a and b is the positive number x that satisfies</p> $\frac{x}{a} = \frac{b}{x}$ <p>So, $x^2 = ab$ and $x = \sqrt{ab}$</p>	<p>Find the geometric mean of</p> <table border="0"> <tr> <td>2 and 8</td> <td>4 and 5</td> <td>8 and 8</td> <td>12 and 6</td> </tr> <tr> <td>$x = \sqrt{2(8)}$</td> <td>$x = \sqrt{4(5)}$</td> <td>$x = \sqrt{8(8)}$</td> <td>$x = \sqrt{12(6)}$</td> </tr> <tr> <td>$x = \sqrt{16}$</td> <td>$x = \sqrt{20}$</td> <td>$x = \sqrt{64}$</td> <td>$x = \sqrt{72}$</td> </tr> <tr> <td>$x = 4$</td> <td>$x = 2\sqrt{5}$</td> <td>$x = 8$</td> <td>$x = 6\sqrt{2}$</td> </tr> </table>	2 and 8	4 and 5	8 and 8	12 and 6	$x = \sqrt{2(8)}$	$x = \sqrt{4(5)}$	$x = \sqrt{8(8)}$	$x = \sqrt{12(6)}$	$x = \sqrt{16}$	$x = \sqrt{20}$	$x = \sqrt{64}$	$x = \sqrt{72}$	$x = 4$	$x = 2\sqrt{5}$	$x = 8$	$x = 6\sqrt{2}$
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$$\frac{4}{2} = \frac{8}{4}$$

$$x = 2\sqrt{5}$$

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